Radiation from Atoms Falling into a Black Hole

Marlan O. Scully[1,2,3], Stephen Fulling[1], David Lee[1], Don Page[4], Wolfgang Schleich[1,5], Anatoly Svidzinsky[1]
[1] *Texas A&M University, College Station, TX 77843;*
[2] *Baylor University, Waco, TX 76798;*
[3] *Princeton University, Princeton, NJ 08544;*
[4] *University of Alberta, Edmonton, Canada T6G 2E1;*
[5] *Universität Ulm, Germany D-89081*
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We show that atoms falling from outside through a cavity into a black hole (BH) emit acceleration radiation which to a distant observer looks much like Hawking BH radiation. In particular, we find the entropy of the acceleration radiation via a simple laser-like analysis. We call this entropy Horizon Brightened Acceleration Radiation (HBAR) entropy to distinguish it from the BH entropy of Bekenstein and Hawking.

I. INTRODUCTION

General relativity as originally developed by Einstein [1] is based on the union of geometry and gravity [2]. Half a century later the union of general relativity and thermodynamics was found to yield surprising results such as Bekenstein-Hawking black hole entropy [3, 4], particle emission from a black hole [4, 5] and acceleration radiation [6]. More recently the connection between black hole (BH) physics and optics, e.g., ultraslow light [7], fiber-optical analog of the event horizon [8] and quantum entanglement [9] has led to fascinating physics.

In their seminal works, Hawking, Unruh and others showed how quantum effects in curved space yield a blend of thermodynamics, quantum field theory and gravity which continues to intrigue and stimulate. For problems as important and startling as Hawking and Unruh radiation, new and alternative approaches are of interest. In that regard it was shown [10] that virtual processes in which atoms jump to an excited state while emitting a photon is an alternative way to view Unruh acceleration radiation. Namely, by breaking and interrupting the virtual processes which take place all around us we can render the virtual photons real.

The present paper is an extension of that logic by considering what happens when atoms fall through the Boulware vacuum [11] into a BH as shown in Fig. 1. The equivalence principle tells us that an atom falling in a gravitational field does not "feel" the effect of gravity, namely its 4−acceleration is equal to zero. However, as we discuss in Appendix A, there is relative acceleration between the atoms and the field modes. This leads to the generation of acceleration radiation. In Appendix B we provide a detailed calculation of the photon emission by atoms falling into a BH.

Specifically we consider an atomic cloud consisting of two level atoms emitting acceleration radiation (see Fig. 1) [10]. We find that the quantum master equation technique, as developed in the quantum theory of the laser [12], provides a useful tool for the analysis of BH acceleration radiation and the associated entropy. In particular, we derive a coarse grained equation of motion for the density matrix of the emitted radiation of the form

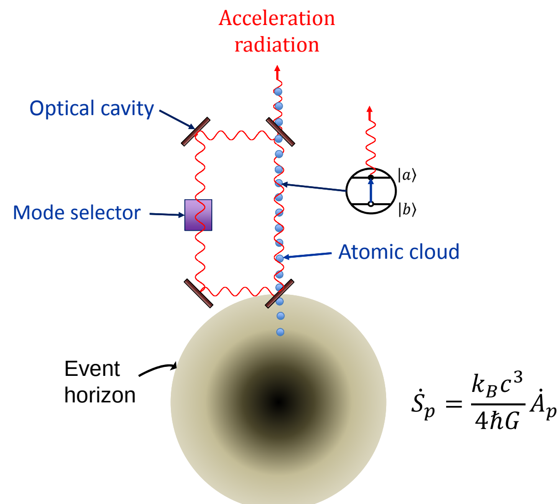

FIG. 1: A BH is bombarded by a pencil-like cloud of two level atoms falling radially from infinity. A cavity is held at the event horizon which shields infalling atoms from the Hawking radiation and the mode selector picks one cavity mode (or a few modes) counterpropagating relative to the atoms. The relative acceleration between the atoms and the field yields generation of acceleration radiation. The physics of the acceleration radiation process corresponds to the excitation of the atom together with the emission of the photon (see Appendix B).

$$\dot{\rho}_{nn} = (\mathcal{M}\rho)_{nn}\,, \tag{1}$$

where the time evolution of the diagonal elements of the density matrix ρ_{nn} is governed by the super operator $\mathcal{M}$ as given by Eq. (7).

Furthermore, we find that once we have cast the acceleration radiation problem in the language of quantum optics and cavity QED the entropy follows directly. Specifically, once we calculate $\dot{\rho}$ for the field produced by accelerating atoms, we can use the von Neumann entropy to write

$$\dot{S}_p = -k_B \mathrm{Tr}(\dot{\rho} \ln \rho) \tag{2}$$

to calculate the radiation entropy flux directly. From the present perspective the acceleration radiation - BH entropy problem is close in spirit to the quantum theory of the laser.

Hawking's pioneering proof that BHs are not black [4] is based on a quantum field theoretic analysis showing that photon emission from a BH is characterized by a temperature T_{BH} and generalized BH entropy. James York [13] gives an analogy between radiation from a BH and total internal reflection in classical optics. He argues that a light beam in a dense medium can undergo total internal reflection at a flat optical surface; but if we sprinkle dust particles on the surface some light will be transmitted. Now the flat surface can be likened to the BH event horizon, the dust is replaced by vacuum fluctuations and light is transmitted through the horizon.

Hawking showed that the radiation that comes out from the BH is described by the temperature

$$T_{\mathrm{BH}} = \frac{\hbar c^3}{8\pi k_B G M}. \tag{3}$$

Hawking then associates the energy of the emitted radiation δE with the loss in energy of the BH $\delta(Mc^2)$ and writes the entropy loss as $\delta S = \delta(Mc^2)/T_{\mathrm{BH}}$. Then using Eq. (3) he obtains

$$\delta S = k_B \frac{8\pi G}{\hbar c} M \delta M = \frac{k_B c^3}{4\hbar G} \delta A, \tag{4}$$

where the BH area in terms of the gravitational radius $r_g = 2GM/c^2$ is given by $A \equiv 4\pi r_g^2 = 16\pi G^2 M^2/c^4$.

In the present paper we analyze the problem of atoms outside the event horizon emitting acceleration radiation as they fall into the BH. The emitted radiation is essentially, but not precisely, thermal and has an entropy analogous to the BH result given by Eq. (4). However, the physics is very different. Here we have radiation coming from the atoms, whereas Hawking radiation requires no extra matter (e.g. atoms). Indeed, Hawking radiation arises just from the BH geometry and the initial state of the quantized field involved.

Historically, Bekenstein [3] introduced the BH entropy concept by information theory arguments. Hawking [4] then introduced the BH temperature to calculate the entropy. In the present approach we calculate the radiation density matrix and then calculate the entropy directly. To distinguish this from the BH entropy we call it the Horizon Brightened Acceleration Radiation (HBAR) entropy.

II. THE HBAR ENTROPY VIA QUANTUM STATISTICAL MECHANICS

As noted earlier, we here consider a BH bombarded by a beam of two-level atoms with transition frequency ω which fall into the event horizon at a rate r (see Fig. 1). The atoms emit and absorb the acceleration radiation.

We seek the density matrix of the field. As in the quantum theory of the laser [12], the (microscopic) change in the density matrix of the field due to any one atom, $\delta\rho^i$, is small. The (macroscopic) change due to ΔN atoms is then

$$\Delta\rho = \sum_i \delta\rho^i = \Delta N \delta\rho. \tag{5}$$

Writing $\Delta N = r\Delta t$, where r is the atomic injection rate, we have the coarse grained equation of motion

$$\frac{\Delta\rho}{\Delta t} = r\delta\rho. \tag{6}$$

We thus obtain an evolution equation for the radiation following the approach used in the quantum theory of the laser [12]. As is further discussed in Appendices B and C, the coarse grained time rate of change of the radiation field density matrix for a particular field mode is found to be

$$\frac{1}{R}\frac{d\rho_{n,n}}{dt} = -\frac{rg^2}{\omega^2} e^{-\xi} \left[(n+1)\rho_{n,n} - n\rho_{n-1,n-1}\right]$$

$$- \frac{rg^2}{\omega^2} e^{\xi} \left[n\rho_{nn} - (n+1)\rho_{n+1,n+1}\right], \tag{7}$$

where g is the atom-field coupling constant, $\xi = 2\pi\nu r_g/c$,

$$R = \frac{\xi}{\sinh(\xi)} \tag{8}$$

and ν is the photon frequency far from the BH. Using Eqs. (2) and (7), we find that the von-Neumann entropy generation rate of the HBAR is (see Appendix D for details)

$$\dot{S}_p = \frac{4\pi k_B r_g}{c} \sum_\nu \dot{\bar{n}}_\nu \nu\,, \tag{9}$$

where $\dot{\bar{n}}_\nu$ is the flux of photons with frequency ν coming from the cavity and propagating away from the BH.

Taking into account that the BH mass change due to photon emission is $\dot{m}_p c^2 = \hbar \sum_\nu \dot{\bar{n}}_\nu \nu$, we arrive at the HBAR entropy/area relation

$$\dot{S}_p = \frac{k_B c^3}{4\hbar G} \dot{A}_p\,. \tag{10}$$

Here $\dot{A}_p = (2\dot{m}_p/M)A$ is the rate of change of the BH area due to photon emission which we are interested in; see Appendix D.

III. DISCUSSION AND SUMMARY

Conversion of virtual photons into directly observable real photons is a subject not without precedent. Moore's accelerating mirrors [14], the rapid change of refractive index considered by Yablonovitch [15] and the more recent observation of the Dynamical Casimir effect in a superconducting circuit [16] are a few examples.

The physics behind acceleration radiation is explained in Ref. [10] (see also [17]) where it is stated that:

> In conclusion our simple model demonstrates that the ground-state atoms accelerated through a field vacuum-state radiate real photons. ... The physical origin of the field energy in the cavity and of the internal energy in the atom is the work done by an external force driving the center-of-mass motion of the atom against the radiation reaction force. Both the present single mode and the many mode effect originate from the transition of the ground-state atom to the excited state with simultaneous emission of photon due to the counter-rotating terms in the Hamiltonian.

In other words the virtual processes in which an atom jumps from the ground state to an excited state, together with the emission of a photon, followed by the reabsorption of the photon and return to the ground state, are altered by the acceleration. The atom is accelerated away from the original point of virtual emission, and there is a small probability that the virtual photon will "get away" before it is reabsorbed as is depicted in Fig. 1.

The Raman effect provides another precedent for the acceleration radiation problem. There are two types of processes taking place in ordinary Raman scattering. (1) The higher frequency pump is absorbed followed by emission of a lower frequency Raman photon. (2) The other process (Fig. 2a) involves the molecule going into a virtual state and at the same time emitting a Raman photon, then a pump photon is absorbed. The excitation of the molecule and emission of photons is thus said to take place before absorption. A similar process occurs in the acceleration radiation and involves, instead of a pump field, a change in the center of mass motion governed by the operators $\hat{c}_p$ and $\hat{c}_q^+$ (Fig. 2b).

Acceleration radiation involves a combination of two effects: acceleration and nonadiabaticity that produce the emitted light. The energy is supplied by the external force field (e.g., the gravitational field of the star).

Gravitational acceleration of atoms is also a source of confusion. The equivalence principle tells us that the atom essentially falls "force free" into the BH. How can it then be radiating? Indeed, the atomic evolution in the atom frame is described by the $e^{i\omega\tau}$ term in the Hamiltonian (B8). From the Hamiltonian we clearly see that it is the photon time (and space) evolution which contain effective acceleration. The radiation modes are fixed relative to the distant stars, and the photons (not the atoms) carry the seed of the acceleration effects in $\hat{V}(\tau)$. The fact that a freely falling atom (detector) is excited and emits radiation is nicely explained in [18, 19].

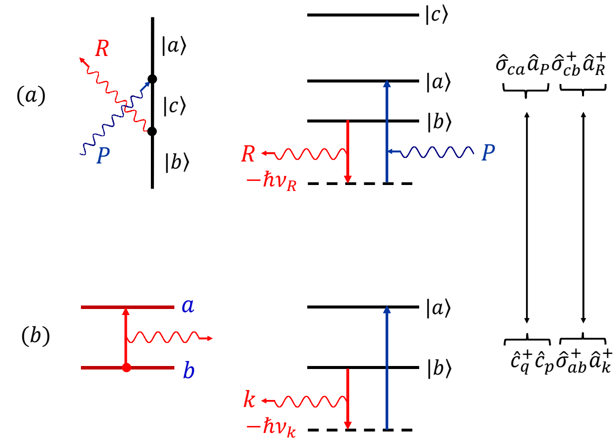

FIG. 2: (a) Processes involved in Raman scattering in which emission occurs before absorption. A molecule promoted from $|b\rangle$ to $|c\rangle$ (virtually) with emission of a Raman photon then absorbs a pump photon while the molecule drops to a state $|a\rangle$. Such a process is due to counter-rotating terms in the Hamiltonian much as in the case of acceleration radiation. (b) Processes involved in acceleration radiation of a two-level atom. Operators $\hat{c}_p$ and $\hat{c}_q^+$ describe the change in the center of mass motion. An analogy between elements of Raman and acceleration radiation processes are shown at the extreme right.

The present model is simple enough to allow a direct calculation of the HBAR entropy. It is a much more tractable problem then the daunting BH entropy issue. It is interesting that the answer for the HBAR entropy we found is essentially the same as the formula for the Bekenstein-Hawking black hole entropy.

Perhaps the quantum master equation approach can provide a useful tool for calculating the latter. That is, by regarding the material inside the horizon as a reservoir (in some sense like the atoms in the present approach), one can perhaps derive an equation of motion for the density matrix of the Hawking radiation along the lines of Eqs. (C1) and (C2) and then calculate the entropy of the Hawking radiation. We plan to address these and other related questions elsewhere.

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Appendix A: Motion of particle in Rindler and Schwarzschild space-time

When atoms are in free fall their operator time dependence in the interaction picture goes as $\hat{\sigma}^+(\tau) = \hat{\sigma}^+(0)e^{i\omega\tau}$, where τ is the proper time of the atom. The corresponding time evolution of the radiation field operator is $\hat{a}_k^+(t) = \hat{a}_k^+(0)\psi\left[t(\tau), z(\tau)\right]$, where $\psi(t,z)$ is the mode function and the space and time parametrization of the field $t(\tau)$ and $z(\tau)$ are to be determined. In what follows we obtain the results in three steps: (1) Special relativity, (2) Rindler metric, (3) Schwarzschild metric.

1. Special Relativity

First of all we note that finding $t(\tau)$ and $z(\tau)$ i.e. the coordinate time and position of the atom in terms of the atom's proper time is really a problem in special relativity. Namely, from the $2D$ Minkowski line element

$$ds^2 = c^2dt^2 - dz^2 \tag{A1}$$

we can write

$$\tau = \int_0^\tau d\tau = \int_0^t \sqrt{1 - \frac{V^2}{c^2}}dt, \tag{A2}$$

where $V = dz/dt$. One can show that for a particle moving with constant proper acceleration a

$$V = \frac{at}{\sqrt{1 + \frac{a^2t^2}{c^2}}} \tag{A3}$$

and, therefore,

$$\tau = \int_0^t \frac{dt}{\sqrt{1 + \frac{a^2t^2}{c^2}}} = \frac{c}{a}\sinh^{-1}\left(\frac{at}{c}\right), \tag{A4}$$

or

$$t(\tau) = \frac{c}{a}\sinh\left(\frac{a\tau}{c}\right). \tag{A5}$$

Likewise, integration of $V(t)$ yields

$$z(t) - z(0) = \int_0^t V(t)dt = \frac{c^2}{a}\left(\sqrt{1 + \frac{a^2t^2}{c^2}} - 1\right). \tag{A6}$$

Setting $z(0) = c^2/a$ and using Eq. (A5) we obtain

$$z(\tau) = \frac{c^2}{a}\cosh\left(\frac{a\tau}{c}\right). \tag{A7}$$

2. Rindler

The Rindler metric for a particle undergoing uniformly accelerated motion is obtained from the Minkowski line element (A1) if we make a coordinate transformation

$$t = \frac{\bar{z}}{c}\sinh\left(\frac{\bar{a}\bar{t}}{c}\right), \tag{A8}$$

$$z = \bar{z}\cosh\left(\frac{\bar{a}\bar{t}}{c}\right), \tag{A9}$$

where $\bar{a}$ is a constant. This leads to the line element

$$ds^2 = \left(\frac{\bar{a}\bar{z}}{c^2}\right)^2 c^2d\bar{t}^2 - d\bar{z}^2, \tag{A10}$$

which is the Rindler line element describing uniformly accelerated motion. Comparison of Eqs. (A8) and (A9) with Eqs. (A5) and (A7) shows that a particle moving along a trajectory with constant $\bar{z}$ in Rindler space has $\tau = \bar{a}\bar{t}/a$ and is uniformly accelerating in Minkowski space with acceleration

$$a = \frac{c^2}{\bar{z}}. \tag{A11}$$

3. Schwarzschild

Finally we make an observation that the $t - r$ part of the Schwarzschild metric,

$$ds^2 = \left(1 - \frac{r_g}{\bar{r}}\right)c^2d\bar{t}^2 - \frac{1}{1 - \frac{r_g}{\bar{r}}}d\bar{r}^2, \tag{A12}$$

where $r_g = 2GM/c^2$ is the gravitational radius, can be approximated around r_g by Rindler space by using the coordinate $0 < \bar{z} \ll r_g$ defined by

$$\bar{r} = r_g + \frac{\bar{z}^2}{4r_g}. \tag{A13}$$

Expanding around r_g

$$1 - \frac{r_g}{\bar{r}} \approx \frac{\bar{z}^2}{4r_g^2} \tag{A14}$$

yields the Rindler metric [20]

$$ds^2 = \frac{\bar{z}^2}{4r_g^2}c^2d\bar{t}^2 - d\bar{z}^2. \tag{A15}$$

According to Eq. (A11), curves of constant $\bar{z}$ (or $\bar{r}$) correspond to uniformly accelerated motions with

$$a = \frac{c^2}{\bar{z}} = \frac{c^2}{2r_g}\frac{1}{\sqrt{1 - \frac{r_g}{\bar{r}}}}. \tag{A16}$$

Appendix B: Acceleration radiation from atoms falling into a black hole

Here we consider a two-level (a is the excited level and b is the ground state) atom with transition angular frequency ω freely falling into a nonrotating BH of mass M along a radial trajectory from infinity with zero initial velocity. We choose the gravitational radius $r_g = 2GM/c^2$

as a unit of distance and r_g/c as a unit of time and introduce the dimensionless distance, time and frequency as

$$r \to r_g r, \quad t \to (r_g/c)t, \quad \omega \to (c/r_g)\omega.$$

In dimensionless Schwarzschild coordinates the atom trajectory is described by the equations

$$\frac{dr}{d\tau} = -\frac{1}{\sqrt{r}}, \quad \frac{dt}{d\tau} = \frac{r}{r-1}, \tag{B1}$$

where t is the dimensionless time in Schwarzschild coordinates and τ is the dimensionless proper time for the atom. Integration of equations (B1) yields

$$\tau = -\frac{2}{3}r^{3/2} + const, \tag{B2}$$

$$t = -\frac{2}{3}r^{3/2} - 2\sqrt{r} - \ln\left(\frac{\sqrt{r}-1}{\sqrt{r}+1}\right) + const. \tag{B3}$$

For a scalar photon in the Regge-Wheeler coordinate

$$r_* = r + \ln(r-1) \tag{B4}$$

the field propagation equation reads

$$\left[\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial r_*^2} + \left(1-\frac{1}{r}\right)\left(\frac{1}{r^3} - \frac{\Delta}{r^2}\right)\right]\psi = 0, \tag{B5}$$

where Δ is the angular part of the Laplacian. We are interested in solutions of this equation outside of the event horizon, that is for $r > 1$. If the dimensionless photon angular frequency $\nu \gg 1$, then the first two terms in Eq. (B5) dominate and one can approximately write

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial r_*^2}\right)\psi = 0. \tag{B6}$$

We consider a solution of this equation describing an outgoing wave

$$\psi = e^{i\nu(t-r_*)} = e^{i\nu[t-r-\ln(r-1)]}, \tag{B7}$$

where ν is the wave frequency measured by a distant observer. In general we will have many modes of the field (frequencies ν) which we will sum over as in Eq. (9).

The interaction Hamiltonian between the atom and the field mode (B7) is

$$\hat{V}(\tau) = \hbar g\left[\hat{a}_\nu e^{-i\nu t(\tau)+i\nu r_*(\tau)} + \text{H.c.}\right]\left(\hat{\sigma}e^{-i\omega\tau} + \text{H.c.}\right), \tag{B8}$$

where the operator $\hat{a}_\nu$ is the photon annihilation operator, $\hat{\sigma}$ is the atomic lowering operator and g is the atom-field coupling constant. We assume that $g \approx$const which is the case for a scalar (spin-0) "photons". Initially the atom is in the ground state and there are no photons for the modes with frequency ν, so that the field is in the Boulware vacuum [11].

The probability of excitation of the atom (frequency ω) with simultaneous emission of a photon with frequency ν is due to a counterrotating term $\hat{a}_\nu^+\hat{\sigma}^+$ in the interaction Hamiltonian. The probability of this event,

$$P_{exc} = \frac{1}{\hbar^2}\left|\int d\tau\, \langle 1_\nu, a|\, \hat{V}(\tau)\, |0, b\rangle\right|^2$$

$$= g^2\left|\int d\tau e^{i\nu t(\tau) - i\nu r_*(\tau)} e^{i\omega\tau}\right|^2,$$

can be written as an integral over the atomic trajectory from $r=\infty$ to the event horizon $r=1$ as

$$P_{exc} = g^2\left|\int_\infty^1 dr\left(\frac{d\tau}{dr}\right)e^{i\nu t(r)-i\nu r_*(r)}e^{i\omega\tau(r)}\right|^2. \tag{B9}$$

Inserting here Eqs. (B2)-(B4) we obtain

$$P_{exc} = g^2\left|\int_1^\infty dr\sqrt{r}e^{-i\nu\left[\frac{2}{3}r^{3/2}+r+2\sqrt{r}+2\ln\left(\sqrt{r}-1\right)\right]}e^{-\frac{2}{3}i\omega r^{3/2}}\right|^2.$$

Making change of the integration variable into $y = r^{3/2}$ yields

$$P_{exc} = \frac{4g^2}{9}\left|\int_1^\infty dy e^{-i\nu\left[\frac{2}{3}y+y^{2/3}+2y^{1/3}+2\ln\left(y^{1/3}-1\right)\right]}e^{-\frac{2}{3}i\omega y}\right|^2. \tag{B10}$$

Next we make another change of the integration variable $x = \frac{2\omega}{3}(y-1)$ and find

$$P_{exc} = \frac{g^2}{\omega^2}\left|\int_0^\infty dx e^{-i\nu\phi(x)}e^{-ix}\right|^2, \tag{B11}$$

where

$$\phi(x) = \frac{x}{\omega} + \left(1+\frac{3x}{2\omega}\right)^{2/3} + 2\left(1+\frac{3x}{2\omega}\right)^{1/3}$$

$$+2\ln\left[\left(1+\frac{3x}{2\omega}\right)^{1/3} - 1\right].$$

The asymptotic behavior of Eq. (B11) at $\omega \gg 1$ can be obtained by expanding the function under the exponential in $1/\omega$. Keeping the leading terms we have

$$\phi(x) \approx 3 + 2\ln\left(\frac{x}{2\omega}\right) + \frac{2x}{\omega}.$$

In the limit $\omega \gg 1$ Eq. (B11) becomes

$$P_{exc} = \frac{g^2}{\omega^2}\left|\int_0^\infty dx e^{-2i\nu\ln x}e^{-ix\left(1+\frac{2\nu}{\omega}\right)}\right|^2$$

$$= \frac{g^2}{\omega^2\left(1+\frac{2\nu}{\omega}\right)^2}\left|\int_0^\infty dx x^{2i\nu}e^{ix}\right|^2. \quad \text{(B12)}$$

Using

$$\int_0^\infty dx x^{2i\nu}e^{ix} = -\frac{\pi e^{-\pi\nu}}{\sinh(2\pi\nu)\,\Gamma(-2i\nu)},$$

where $\Gamma(z)$ is the gamma function, and the property $|\Gamma(-ix)|^2 = \pi/[x\sinh(\pi x)]$ we find

$$P_{exc} = \frac{4\pi g^2\nu}{\omega^2\left(1+\frac{2\nu}{\omega}\right)^2}\frac{1}{e^{4\pi\nu}-1}. \quad \text{(B13)}$$

P_{exc} becomes exponentially small for $\nu \gg 1$. Thus, acceleration radiation will not be emitted with very large ν. On the other hand, typical atomic frequencies $\omega \gg 1$ and, therefore, in the following one can assume that $\omega \gg \nu$. Then, in the dimensional units Eq. (B13) reads

$$P_{exc} = \frac{4\pi g^2 r_g\nu}{c\omega^2}\frac{1}{e^{\frac{4\pi r_g\nu}{c}}-1}. \quad \text{(B14)}$$

The probability of photon absorption is obtained by changing $\nu \to -\nu$, which for $\omega \gg \nu$ yields

$$P_{abs} = e^{\frac{4\pi r_g\nu}{c}}P_{exc}. \quad \text{(B15)}$$

Appendix C: Density matrix for the field mode

The (microscopic) change in the density matrix of a field mode $\delta\rho^i$ due to an atom injected at time τ_i is

$$\delta\rho^i = -\frac{1}{\hbar^2}\int_{\tau_i}^{\tau_i+T_{\rm int}}\int_{\tau_i}^{\tau_i+\tau'} d\tau' d\tau''$$

$$\mathrm{Tr}_{atom}\left[\hat{V}(\tau'),\left[\hat{V}(\tau''),\rho^{\rm atom}(\tau_i)\otimes\rho(t(\tau_i))\right]\right], \quad \text{(C1)}$$

where $T_{\rm int}$ is the proper atom-field interaction time, Tr_{atom} denotes the trace over atom states and $\hat{V}(\tau)$ is the interaction Hamiltonian between the atom and the field mode given by Eq. (B8). The time τ is the atomic proper time, i.e., the time measured by an observer riding along with the atom.

In the case of random injection times, the equation of motion for the density matrix of the field is

$$\frac{d\rho_{n,n}}{dt} = -\Gamma_e\left[(n+1)\rho_{n,n} - n\rho_{n-1,n-1}\right] -$$

$$-\Gamma_a\left[n\rho_{n,n} - (n+1)\rho_{n+1,n+1}\right], \quad \text{(C2)}$$

where Γ_e and Γ_a are emission and absorption rates of the photon in the cavity, $\Gamma_{e,a} = r|gI_{e,a}|^2$, and $I_{e,a}$ are given by the integrals

$$ge^{-i\xi/\pi}I_{e,a} = -\frac{i}{\hbar}\int_{\tau_i}^{\tau_i+T_{\rm int}} V_{e,a}d\tau,$$

where $\xi = 2\pi\nu r_g/c$ and ν is the mode frequency far from BH. We note that the absorption and emission matrix elements of the interaction Hamiltonian are as in Appendix B

$$V_a = \langle 0,a|\,\hat{V}(\tau)\,|1,b\rangle\,, \quad V_e = \langle 1,a|\,\hat{V}(\tau)\,|0,b\rangle\,,$$

and obtain Eq. (C2). Leakage of photons into "outer space" relative to the atomic cloud-BH complex as in Fig. 1 can be taken into account by adding "leakage" terms to the density matrix equation (C2). However, if the rate of photon loss from the cavity is much smaller then Γ_a, such terms can be omitted in Eq. (C2).

When absorption is greater then emission there is a steady state solution of Eq. (C2) given by the thermal distribution [12]

$$\rho^{S.S.}_{n,n} = \exp(-2\xi n)\left[1-\exp(-2\xi)\right]. \quad \text{(C3)}$$

In order to approach this steady state solution, we need a cavity to restrict the modes to a finite range of the Regge-Wheeler coordinate r_*, so the bottom of the cavity must be at $r_b > r_g$, and the top must be at $r_t < \infty$. This will modify the analysis of Appendix B, but we can then take the limit as $r_b \to r_g$ and $r_t \to \infty$.

Appendix D: Entropy flux

The time rate of change of entropy inside the cavity due to photon generation,

$$\dot{S}_p = -k_B\sum_{n,\nu}\dot{\rho}_{n,n}\ln\rho_{n,n}, \quad \text{(D1)}$$

to a good approximation can be written as

$$\dot{S}_p \approx -k_B\sum_{n,\nu}\dot{\rho}_{n,n}\ln\rho^{S.S.}_{n,n} \quad \text{(D2)}$$

once one has approached the steady state solution [21]. The steady state density matrix $\rho^{S.S.}_{n,n}$ is given by Eq. (C3). Inserting it into (D2) gives

$$\dot{S}_p \approx \frac{4\pi k_B r_g}{c}\sum_\nu \dot{\bar{n}}_\nu\nu, \quad \text{(D3)}$$

where $\dot{\bar{n}}_\nu$ is the photon flux from the cavity.

Recalling the BH area $A \equiv 4\pi r_g^2$, where the gravitational radius $r_g = 2MG/c^2$, and $\dot{m}_p c^2 = \hbar\sum_\nu\dot{\bar{n}}_\nu\nu$ is the power carried away by the emitted photons, we arrive at the HBAR entropy/area relation

$$\dot{S}_p = \frac{k_B c^3}{4\hbar G}\dot{A}_p\,. \quad \text{(D4)}$$

Here $\dot{A}_p = 32\pi G^2 M\dot{m}_p/c^4$ is the rate of change of the BH area due to photon emission. The BH rest mass changes as $\dot{M} = \dot{m}_{\rm atom} + \dot{m}_p$ due to the atomic cloud adding to and the emitted photons taking from the mass of the BH. The BH area A is proportional to M^2 and, hence, $\dot{A} = (2\dot{M}/M)A = \dot{A}_{\rm atom} + \dot{A}_p$.

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